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Solar system and equivalence principle constraints on $f(R)$ gravity by chameleon approach

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We study constraints on $f(R)$ dark energy models from solar system experiments combined with experiments on the violation of equivalence principle. When the mass of an equivalent scalar field degree of freedom is heavy in a region with high density, a spherically symmetric body has a thin-shell so that an effective coupling of the fifth force is suppressed through a chameleon mechanism. We place experimental bounds on the cosmologically viable models recently proposed in literature which have an asymptotic form $f(R) = R - \lambda R_c[1 - (R_c/R)^{2n}]$ in the regime $R \gg R_c$. From the solar-system constraints on the post-Newtonian parameter γ, we derive the bound $n > 0.5$, whereas the constraints from the violations of weak and strong equivalence principles give the bound $n > 0.9$. This allows a possibility to find the deviation from the ΛCDM cosmological model. For the model $f(R) = R - \lambda R_c(R/R_c)^p$ with $0 < p < 1$ the severest constraint is found to be $p < 10^{-10}$, which shows that this model is hardly distinguishable from the ΛCDM cosmology.

The recent data coming from the luminosity distance of Supernovae Ia [1], the wide galaxy surveys [2] and the anisotropy of Cosmic Microwave Background [3] suggest that about 70 % of the energy density of the present universe is composed by dark energy responsible for an accelerated expansion. The cosmological constant is the most relevant candidate to interpret the cosmic expansion, but, in order to overcome its intrinsic shortcomings associated with the energy scale, several alternative models such as quintessence and k-essence have been proposed (see Ref. [4] for reviews). Most of these models have the common feature to introduce new sources into the cosmological dynamics, but, from an "economic" point of view, it would be preferable to develop scenarios consistent with observations without invoking extra parameters or components non-testable at a fundamental level.

The simplest extension to the ΛCDM model is presumably the so called $f(R)$ gravity, where $f(R)$ is a general function of the Ricci scalar R [5] (see Ref. [6] for an early work). In Ref. [7] the authors derived the conditions under which a successful sequence of radiation, matter and accelerated epochs can be realized. In addition the stability conditions $f_{,R} > 0$ and $f_{,RR} > 0$ are required to avoid ghosts and tachyons for $R \geq R_1$, where R_1 is the Ricci scalar at a de-Sitter point [8]. There exist viable $f(R)$ models that can satisfy both background cosmological constraints and stability conditions [8, 9, 10, 11, 12, 13, 14, 15, 16]. These models can satisfy solar system constraints under a chameleon mechanism, that is, a nonlinear effect arising from a large departure from the background value of R [11, 13, 14, 16]. In this brief report, we place constraints on viable $f(R)$ gravity models under the chameleon mechanism [17] by using both solar-system and equivalence principle bounds.

We start with the following action in $f(R)$ gravity:

$$S = \int \mathrm{d}^4x\sqrt{-g}\, f(R)/2 + S_m(g_{\mu\nu}, \Psi_m)\,, \qquad (1)$$

where S_m is a matter Lagrangian that depends on the metric $g_{\mu\nu}$ and matter fields Ψ_m. We use the unit $M_{\rm pl}^2 = (8\pi G)^{-1} = 1$, where $M_{\rm pl}$ and G are a reduced Planck mass and a bare gravitational constant respectively.

We introduce a new metric variable $\tilde{g}_{\mu\nu}$ and a scalar field ϕ, as

$$\tilde{g}_{\mu\nu} = \psi g_{\mu\nu}\,, \quad \phi = \sqrt{3/2}\ln\psi\,, \qquad (2)$$

where $\psi = \partial f/\partial R$. Then the action in the Einstein frame is given by [18]

$$\begin{aligned} S \;=\; & \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\tilde{R}/2 - (\tilde{\nabla}\phi)^2/2 - V(\phi)\right] \\ & + S_m(\tilde{g}_{\mu\nu}e^{2\beta\phi}, \Psi_m)\,, \end{aligned} \qquad (3)$$

where

$$\beta = -\frac{1}{\sqrt{6}}\,, \quad V = \frac{R(\psi)\psi - f}{2\psi^2}\,. \qquad (4)$$

The field ϕ is directly coupled to a non-relativistic matter with a constant coupling β.

In a spherically symmetric spacetime, the variation of the action (3) with respect to the scalar field ϕ gives

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\tilde{r}^2} + \frac{2}{\tilde{r}}\frac{\mathrm{d}\phi}{\mathrm{d}\tilde{r}} = \frac{\mathrm{d}V_{\rm eff}}{\mathrm{d}\phi}\,, \qquad (5)$$

where $\tilde{r}$ is the distance from the center of symmetry and

$$V_{\rm eff}(\phi) = V(\phi) + e^{\beta\phi}\rho^*\,. \qquad (6)$$

Here ρ^* is a conserved quantity in the Einstein frame [17], which is related with the energy density ρ in the Jordan frame via the relation $\rho^* = e^{3\beta\phi}\rho$.

We assume that a spherically symmetric body has a constant density $\rho^* = \rho^*_A$ inside the body ($\tilde{r} < \tilde{r}_c$) and that the energy density outside the body ($\tilde{r} > \tilde{r}_c$) is

$\rho^* = \rho_B^*$. The mass M_c of the body and the gravitational potential Φ_c at the radius $\tilde{r}_c$ are given by $M_c = (4\pi/3)\tilde{r}_c^3\rho_A^*$ and $\Phi_c = M_c/8\pi\tilde{r}_c$, respectively. The effective potential $V_{\rm eff}(\phi)$ has two minima at the field values ϕ_A and ϕ_B satisfying $V'_{\rm eff}(\phi_A) = 0$ and $V'_{\rm eff}(\phi_B) = 0$, respectively. The former corresponds to the region with a high density that gives rise to a heavy mass squared $m_A^2 \equiv V''_{\rm eff}(\phi_A)$, whereas the latter to the lower density region with a lighter mass squared $m_B^2 \equiv V''_{\rm eff}(\phi_B)$.

In the high-density regime with a heavy field mass, it is known that the spherically symmetric body has a thin-shell under the chameleon mechanism. When the thin-shell develops inside the body, the following thin-shell parameter is much smaller than the order of unity [17]:

$$\frac{\Delta\tilde{r}_c}{\tilde{r}_c} = \frac{\phi_B - \phi_A}{6\beta\Phi_c}\,. \tag{7}$$

Solving Eq. (5) with appropriate boundary conditions, the field profile outside the body ($\tilde{r} > \tilde{r}_c$) is given by [17]

$$\phi(\tilde{r}) \simeq -\frac{\beta_{\rm eff}}{4\pi}\frac{M_c e^{-m_B(\tilde{r}-\tilde{r}_c)}}{\tilde{r}} + \phi_B\,, \tag{8}$$

where the magnitude of the effective coupling, $\beta_{\rm eff} = (3\beta)(\Delta\tilde{r}_c/\tilde{r}_c)$, is much smaller than unity when the thin-shell is formed.

Let us study concrete $f(R)$ models that can satisfy local gravity constraints as well as cosmological and stability conditions. Hu and Sawicki [11] proposed the following model

$$f(R) = R - \lambda R_c \frac{(R/R_c)^{2n}}{(R/R_c)^{2n}+1}\,, \tag{9}$$

whereas Starobinsky [8] proposed another viable model

$$f(R) = R - \lambda R_c \left[1 - \left(1 + R^2/R_c^2\right)^{-n}\right]\,. \tag{10}$$

In both models n, λ and R_c are positive constants. Since $f(R=0) = 0$, the cosmological constant disappears in a flat spacetime. Other $f(R)$ models with similar features have been discussed in Refs. [12, 13, 15]. In these models a de-Sitter point responsible for the late-time acceleration exists at $R = R_1$ (> 0), where R_1 is derived by solving the equation $R_1 f_{,R}(R_1) = 2f(R_1)$ [7]. Note that R_c is not much different from the present cosmological density $\rho_c \simeq 10^{-29}\,{\rm g/cm^3}$.

In the region $R \gg R_c$ both models (9) and (10) behave as

$$f(R) \simeq R - \lambda R_c \left[1 - (R_c/R)^{2n}\right]\,. \tag{11}$$

Inside and outside the spherically symmetric body the effective potential (6) has minima at $\phi_A \simeq -\sqrt{6}n\lambda(R_c/\rho_A)^{2n+1}$ and $\phi_B \simeq -\sqrt{6}n\lambda(R_c/\rho_B)^{2n+1}$, respectively. Since $\rho_A \gg \rho_B \gg \rho_c$ one has $|\phi_A| \ll |\phi_B| \ll 1$ and $\tilde{r} \simeq r$, provided that n and λ are not much different from the order of unity. In the following we omit the tilde for the quantity r. From Eq. (7) the thin-shell parameter is approximately given by

$$\frac{\Delta r_c}{r_c} \simeq n\lambda\left(\frac{R_c}{\rho_B}\right)^{2n+1}\frac{1}{\Phi_c}\,. \tag{12}$$

Let us first discuss post-Newtonian solar-system constraints on the model (11). In the weak-field approximation the spherically symmetric metric in the Jordan frame is

$${\rm d}s^2 = -[1-2\mathcal{A}(r)]{\rm d}t^2 + [1+2\mathcal{B}(r)]{\rm d}r^2 + r^2{\rm d}\Omega^2\,, \tag{13}$$

where $\mathcal{A}(r)$ and $\mathcal{B}(r)$ are the functions of r. It was shown in Ref. [19] that under the chameleon mechanism the post-Newton parameter, $\gamma = \mathcal{B}(r)/\mathcal{A}(r)$, is approximately given by

$$\gamma \simeq \frac{1-\Delta r_c/r_c}{1+\Delta r_c/r_c}\,, \tag{14}$$

provided that the condition $m_B r \ll 1$ holds on solar-system scales. The present tightest constraint on γ is $|\gamma - 1| < 2.3\times10^{-5}$ [20], which translates into

$$\frac{\Delta r_c}{r_c} < 1.15\times10^{-5}\,. \tag{15}$$

For the model (11) the de-Sitter point corresponds to $\lambda = x_1^{2n+1}/(2(x_1^{2n}-n-1))$, where $x_1 = R_1/R_c$. Using this relation together with $\Phi_c \simeq 2.12\times10^{-6}$ for the Sun, the bound (15) leads to

$$\frac{n}{2(x_1^{2n}-n-1)}\left(\frac{R_1}{\rho_B}\right)^{2n+1} < 2.4\times10^{-11}\,. \tag{16}$$

For the stability of the de-Sitter point we require that $m = Rf_{,RR}/f_{,R} < 1$ at $R = R_1$ [7], which gives the condition $x_1^{2n} > 2n^2+3n+1$. Hence the term $n/2(x_1^{2n}-n-1)$ in Eq. (16) is smaller than 0.25 for $n > 0$. Assuming that R_1 and ρ_B are of the orders of the present cosmological density $10^{-29}\,{\rm g/cm^3}$ and the baryonic/dark matter density $10^{-24}\,{\rm g/cm^3}$ in our galaxy, respectively, we obtain the constraint

$$n > 0.5\,. \tag{17}$$

Thus n does not need to be much larger than unity. Hu and Sawicki derived the Ricci scalar R as a function of r by considering the density profile of the Sun. While we have obtained the bound (17) without taking into account such modifications, this bound is consistent with the one derived by Hu and Sawicki (see Eq. (67) in Ref. [11]).

Let us also study the models of the type [9, 10, 19]

$$f(R) = R - \lambda R_c (R/R_c)^p\,, \quad 0 < p < 1\,, \tag{18}$$

where λ and R_c are positive constants. We do not consider the models with negative p, because they suffer from instability problems of perturbations associated

with negative $f_{,RR}$ [21, 22] as well as the absence of the matter-dominated epoch [23]. In this case the field ϕ_B is given by $\phi_B = -(\sqrt{6}/2)\lambda p(R_c/\rho_B)^{1-p}$. Since the de-Sitter point, $x_1 = R_1/R_c$, satisfies the relation $\lambda = x_1^{1-p}/(2-p)$, the bound (15) translates into

$$\frac{p}{2-p}\left(\frac{R_1}{\rho_B}\right)^{1-p} < 4.9\times 10^{-11}\,. \tag{19}$$

Taking $R_1 = \rho_1 = 10^{-29}$ g/cm^3 and $\rho_B = 10^{-24}$ g/cm^3, we obtain the constraint

$$p < 5\times 10^{-6}\,. \tag{20}$$

Hence the deviation from the ΛCDM model is very small.

Let us next place experimental bounds from a possible violation of the equivalence principle (EP). In doing so we shall discuss the thin-shell condition around the Earth under the chameleon mechanism [17]. The Earth has a radius $r_\oplus = 6\times 10^3$ km with a mean density $\rho_\oplus \simeq 5.5$ g/cm^3. The atmosphere exists in the region $r_\oplus < r < r_{\rm atm}$ with a homogeneous density $\rho_{\rm atm} \simeq 10^{-3}$ g/cm^3. The region outside the atmosphere ($r > r_{\rm atm}$) has a homogenous density $\rho_G \simeq 10^{-24}$ g/cm^3. Defining the gravitational potentials as $\Phi_\oplus = \rho_\oplus r_\oplus^2/6$ and $\Phi_{\rm atm} = \rho_{\rm atm} r_{\rm atm}^2/6$, we have that $\Phi_\oplus \simeq 5.5\times 10^3 \Phi_{\rm atm}$ because $\rho_\oplus \simeq 5.5\times 10^3 \rho_{\rm atm}$ and $r_\oplus \simeq r_{\rm atm}$. Recalling the relation $\Delta r_{\rm atm}/r_{\rm atm} = (\phi_G - \phi_{\rm atm})/(6\beta\Phi_{\rm atm})$, where ϕ_G and $\phi_{\rm atm}$ correspond to the field values at the local minima of the effective potential (6) in the regions $r > r_{\rm atm}$ and $r_\oplus < r < r_{\rm atm}$ respectively, we find $\Delta r_\oplus/r_\oplus \equiv -(\phi_G - \phi_{\rm atm})/\sqrt{6}\Phi_\oplus \simeq 2.0\times 10^{-4}(\Delta r_{\rm atm}/r_{\rm atm})$.

When the atmosphere has a thin-shell then the thickness of the shell ($\Delta r_{\rm atm}$) is smaller than that of the atmosphere: r_s = 10-10^2 km. Taking the value $r_s = 10^2$ km and $r_{\rm atm} = 6.5\times 10^3$ km, we obtain $\Delta r_{\rm atm}/r_{\rm atm} < 1.6\times 10^{-2}$. Hence the condition for the atmosphere to have a thin-shell is estimated as

$$\frac{\Delta r_\oplus}{r_\oplus} \lesssim 10^{-6}\,. \tag{21}$$

Let us discuss solar system tests of EP that makes use of the free-fall acceleration of the Moon and the Earth toward the Sun. The constraint on the difference of two accelerations is given by

$$\eta \equiv 2\frac{|a_{\rm Moon} - a_\oplus|}{a_{\rm Moon} + a_\oplus} < 10^{-13}\,. \tag{22}$$

The Sun and the Moon have the thin-shells like the Earth [17], in which case the field profiles outside the spheres are given as in Eq. (8) with the replacement of corresponding quantities. We note that the acceleration induced by a fifth force with the field profile $\phi(r)$ and the effective coupling $\beta_{\rm eff}$ is $a^{\rm fifth} = |\beta_{\rm eff}\phi(r)|$. Then the accelerations $a_\oplus$ and $a_{\rm Moon}$ are [17]

$$a_\oplus \simeq \frac{GM_\odot}{r^2}\left[1 + 3\left(\frac{\Delta r_\oplus}{r_\oplus}\right)^2 \frac{\Phi_\oplus}{\Phi_\odot}\right]\,, \tag{23}$$

$$a_{\rm Moon} \simeq \frac{GM_\odot}{r^2}\left[1 + 3\left(\frac{\Delta r_\oplus}{r_\oplus}\right)^2 \frac{\Phi_\oplus^2}{\Phi_\odot\Phi_{\rm Moon}}\right]\,, \tag{24}$$

where $\Phi_\odot \simeq 2.1\times 10^{-6}$, $\Phi_\oplus \simeq 7.0\times 10^{-10}$ and $\Phi_{\rm Moon} \simeq 3.1\times 10^{-11}$ are the gravitational potentials of Sun, Earth and Moon, respectively. Hence the condition (22) translates into

$$\frac{\Delta r_\oplus}{r_\oplus} < 2\times 10^{-6}\,, \tag{25}$$

which gives the same order of the upper bound as in the thin-shell condition (21) for the atmosphere. The constraint coming from the violation of strong equivalence principle [20] provides a bound $\Delta r_\oplus/r_\oplus < 10^{-4}$ [17], which is weaker than (25).

Let us derive constraints on the models (9) and (10) under the bound (25). On using the relation $|\phi_G| = \sqrt{6}n\lambda(R_c/\rho_G)^{2n+1} \gg |\phi_{\rm atm}|$, we obtain

$$n\lambda\left(\frac{R_c}{\rho_G}\right)^{2n+1} < 10^{-15}\,. \tag{26}$$

Taking the similar procedure we have taken to reach Eq. (17) from Eq. (16), we find the following constraint

$$n > 0.9\,. \tag{27}$$

This is stronger than the bound (17) derived from post-Newtonian tests in the solar system.

In the model (18) the bound (25) leads to $\lambda p\,(R_c/\rho_G)^{1-p} < 10^{-15}$, which gives the constraint

$$p < 10^{-10}\,. \tag{28}$$

Thus the model is required to be very close to the ΛCDM model to satisfy the condition (25).

Let us next discuss constraints from fifth force experiments that are carried out in a vacuum [20]. Modeling a vacuum chamber as a sphere with radius $r_{\rm vac}$, the energy density is given by $\rho(r) = 0$ for $r < r_{\rm vac}$ and $\rho(r) = \rho_{\rm atm}$ for $r > r_{\rm vac}$. Inside the chamber we consider two identical bodies of uniform density ρ_c, radius r_c and total mass M_c. If these bodies have thin-shells, their field profiles are given by

$$\phi(r) = -\frac{\beta_{\rm eff}}{4\pi}\frac{M_c e^{-r/r_{\rm vac}}}{r} + \phi_{\rm vac}\,, \tag{29}$$

where $\phi_{\rm vac}$ is the field value when the mass squared of the field balances with the curvature $r_{\rm vac}^{-2}$ of the chamber. In Eq. (29) we used the fact that the interaction range m_B^{-1} outside the bodies is of the order of $r_{\rm vac}$ [17]. The laboratory experiment constrains the coupling to be $2\beta_{\rm eff}^2 < 10^{-3}$ [20], which translates into the condition

$$\frac{\Delta r_c}{r_c} < 1.7\times 10^{-2}\,. \tag{30}$$

Thus it is crucial to have thin-shells to satisfy the experimental bound.

We have $\Delta r_c/r_c \simeq -\phi_{\rm vac}/\sqrt{6}\Phi_c$ under the condition that $|\phi_{\rm vac}|$ is much larger than the field value $|\phi_A|$ inside the bodies. A typical test body used in Hoskins *et al.* has a mass $M_c \sim 40$ g and a radius $r_c \sim 1\,{\rm cm}$ [20]. Hence the bound (30) translates into

$$|\phi_{\rm vac}| < 10^{-28}\,. \tag{31}$$

For the models (9) and (10) we obtain $\phi_{\rm vac}$ in the region $R \gg R_c$:

$$\phi_{\rm vac} = -\sqrt{6}n\lambda \left[\frac{R_c r_{\rm vac}^2}{6n(2n+1)\lambda}\right]^{\frac{2n+1}{2n+2}}\,. \tag{32}$$

Then the constraint (31) gives

$$C\left(r_{\rm vac}/R_1^{-1/2}\right)^{\frac{2n+1}{n+1}} < 10^{-28}\,, \tag{33}$$

where $C \equiv \sqrt{6}n\lambda\left[\frac{1}{6n(2n+1)\lambda}\frac{R_c}{R_1}\right]^{\frac{2n+1}{2n+2}}$. From the relation $\lambda = x_1^{2n+1}/(2(x_1^{2n}-n-1))$ we find that C is not larger than the order of 0.1. Using $R_1^{-1/2} \sim H_0^{-1} \sim 10^{28}$ cm, we get the following constraint:

$$n > 0\,. \tag{34}$$

This is much weaker than the bounds (17) and (27).

For the model (18) the field value $\phi_{\rm vac}$ is given by

$$\phi_{\rm vac} = -\frac{\sqrt{6}}{2}\lambda p\left[\frac{R_c r_{\rm vac}^2}{3\lambda p(1-p)}\right]^{\frac{1-p}{2-p}}\,. \tag{35}$$

Making use of the relation $\lambda = x_1^{1-p}/(2-p)$ at the de-Sitter point, the condition (31) gives the bound

$$p < 1.5\times 10^{-2}\,. \tag{36}$$

Again this is much weaker than the bounds (20) and (28).

In summary we have found that the models (9) and (10) are consistent with the present local gravity experiments for $n > 0.9$, whereas the model (18) is hardly distinguishable from the ΛCDM cosmology because of the constraint $p < 10^{-10}$. These bounds are stronger than those derived by post-Newtonian tests in the solar system and are the main results of our paper. The models (9) and (10) allow the possibility to show appreciable deviations from the ΛCDM model cosmologically around the present epoch [8, 11, 13, 16]. It will be certainly of interest to find some signatures of modified gravity in future high-precision local gravity experiments such as the STEP [24] or GAIA [25] satellites as well as in cosmological observations [26] such as the galaxy power spectrum, Cosmic Microwave Background and weak lensing.

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[1] A. G. Riess *et al.* Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.* Astrophys. J **517**, 565 (1999).
[2] S. Cole *et al.*, Mon. Not. Roy. Astron. Soc. **362**, 505 (2005); M. Tegmark *et al.*, Phys. Rev. D **74**, 123507 (2006).
[3] D. N. Spergel *et al.* Astrophys. J. Suppl. **148**, 175 (2003); D. N. Spergel *et al.* ApJS **170**, 377 (2007).
[4] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000); S. M. Carroll, Living Rev. Rel. **4**, 1 (2001); T. Padmanabhan, Phys. Rept. **380**, 235 (2003); P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006); S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007); S. Capozziello and M. Francaviglia, Gen. Rel. Grav. **40**, 357 (2008).
[5] S. Capozziello, Int. J. Mod. Phys. **D 11**, 483, (2002); S. Capozziello, S. Carloni and A. Troisi, Rec. Res. Develop. Astron. Astrophys. **1**, 625 (2003), arXiv:astro-ph/0303041; S. Capozziello, V. F. Cardone, S. Carloni and A. Troisi, Int. J. Mod. Phys. **D**, 12, 1969 (2003); S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D **70**, 043528 (2004); S. Nojiri and S. D. Odintsov, Phys. Lett. **B 576**, 5 (2003); Phys. Rev. D **68**, 123512 (2003).
[6] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).
[7] L. Amendola, R. Gannouji, D. Polarski and S. Tsujikawa, Phys. Rev. D **75**, 083504 (2007).
[8] A. A. Starobinsky, JETP Lett. **86**, 157 (2007).
[9] B. Li and J. D. Barrow, Phys. Rev. D **75**, 084010 (2007).
[10] L. Amendola and S. Tsujikawa, Phys. Lett. B **660**, 125 (2008).
[11] W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007).
[12] S. A. Appleby and R. A. Battye, Phys. Lett. B **654**, 7 (2007).
[13] S. Tsujikawa, Phys. Rev. D **77**, 023507 (2008).
[14] I. Navarro and K. Van Acoleyen, JCAP **0702**, 022 (2007).
[15] S. Nojiri and S. D. Odintsov, Phys. Lett. B **652**, 343 (2007).
[16] S. Tsujikawa, K. Uddin and R. Tavakol, Phys. Rev. D **77**, 043007 (2008).
[17] J. Khoury and A. Weltman, Phys. Rev. Lett. **93**, 171104 (2004); Phys. Rev. D **69**, 044026 (2004).
[18] K. i. Maeda, Phys. Rev. D **39**, 3159 (1989).
[19] T. Faulkner, M. Tegmark, E. F. Bunn and Y. Mao, Phys. Rev. D **76**, 063505 (2007).
[20] C. M. Will, Living Rev. Relativity **9** (2006), arXiv:gr-qc/0510072.
[21] A. D. Dolgov and M. Kawasaki, Phys. Lett. B **573**, 1 (2003).

[22] S. M. Carroll, I. Sawicki, A. Silvestri and M. Trodden, New J. Phys. **8**, 323 (2006); R. Bean, D. Bernat, L. Pogosian, A. Silvestri and M. Trodden, Phys. Rev. **D 75**, 064020 (2007); Y. S. Song, W. Hu and I. Sawicki, Phys. Rev. **D 75**, 044004 (2007); I. Sawicki and W. Hu, Phys. Rev. D **75**, 127502 (2007).

[23] L. Amendola, D. Polarski and S. Tsujikawa, Phys. Rev. Lett. **98**, 131302 (2007); Int. J. Mod. Phys. D **16**, 1555 (2007).

[24] J. Mester *et al.*, Class. Quant. Grav. **18**, 2475 (2001).

[25] A. Vecchiato *et al.*, Astron. Astrophys. **399**, 337 (2003).

[26] M. Amarzguioui, O. Elgaroy, D. F. Mota and T. Multamaki, Astron. Astrophys. **454**, 707 (2006); Y. S. Song, H. Peiris and W. Hu, Phys. Rev. D **76**, 063517 (2007); S. Fay, R. Tavakol and S. Tsujikawa, Phys. Rev. D **75**, 063509 (2007); L. Pogosian and A. Silvestri, arXiv:0709.0296 [astro-ph]; A. De Felice, P. Mukherjee and Y. Wang, Phys. Rev. D **77**, 024017 (2008).